MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

BMS1024 – MANAGERIAL STATISTICS

(All sections / Groups)

02 JUNE 2017
3.00 p.m – 5.00 p.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **THIRTEEN (13)** printed pages with:
   Section A: Ten (10) multiple choice questions (20%)
   Section B: Three (3) structured questions (80%)

2. Answer **ALL** questions.

3. Answer **Section A** in the multiple-choice answer sheet provided and **Section B** in the answer booklet provided.

4. Statistical tables are attached at the end of the question paper.

5. Students are allowed to use non-programmable scientific calculators with no restrictions.

SECTION A: MULTIPLE CHOICE QUESTIONS (20 MARKS)

There are TEN (10) questions in this section. Answer ALL questions on the multiple choice answer sheet.

1. Which one of the following variables is not qualitative?
   A. Age of a person
   B. Gender of a person
   C. Choice on a test item; true or false
   D. Marital status of a person

2. _____ is used to compare the variation or dispersion in two or more sets of data even though they are measured in different units.
   A. Range
   B. Standard deviation
   C. Coefficient of variation
   D. Interquartile range

3. In probability theories, events which can never occur together are classified as
   A. Collectively exclusive events
   B. Mutually exhaustive events
   C. Mutually exclusive events
   D. Independent events

4. The sampling distribution of X appears as below:

X	10	15	20	25	30
P(X)	0.25	0.30	0.10	0.3	x

   Then, the expected value of X, $E(X)$ is
   A. 17.75
   B. 18
   C. 17.5
   D. Cannot be determined

5. A study shows that during weekend, the average number of cars queuing at a service station petrol pump in a garage is four. What is the probability that during a particular weekend, there are less than 2 cars were queuing?
   A. 0.0183
   B. 0.2198
   C. 0.2385
   D. 0.0920

Continued...

6. The owner of a fish market determined that the average weight for a catfish is 3.2 pounds with a standard deviation of 0.8 pound. A citation catfish should be one of the top 2% in weight. Assuming the weights of catfish are normally distributed, at what weight (in pounds) should the citation designation be established?
   A. 1.56 pounds
   B. 4.84 pounds
   C. 3.2 pounds
   D. 8.44 pounds

7. A major department store chain is interested in estimating the mean amount its credit card customers spent on their first visit to the chain's new store in the mall. Fifteen credit card accounts were randomly sampled and analyzed with the following results: $\bar{x} = RM50.50$ and s = RM 20. Assuming the distribution of the amount spent on their first visit is normal, what is the shape of the sampling distribution of the sample mean that will be used to create the desired confidence interval for μ?
   A. approximately normal with a mean of RM 50.50
   B. a standard normal distribution
   C. a *t*-distribution with 15 degrees of freedom
   D. a *t*-distribution with 14 degrees of freedom

8. A real estate company is interested in testing whether the mean time that a family in Gotham has been living in their current homes is less than a family in Metropolis. A table is provided as below to display data on length of residence in each city:

	Gotham	Metropolis
Sample Mean	35 months	50 months
Population standard deviation	900 months	1050 months
Sample observations	100 families	150 families

Determine the value of the test statistic for this data.
   A. -0.1207
   B. 100
   C. 1050
   D. 35

9. If an economist wishes to determine whether there is evidence that mean family income in a community exceeds RM50,000,
   A. a one tail test should be utilized.
   B. a two tail test should be utilized.
   C. either a one tail or two tail test could be applied with equivalent results.
   D. None of the above.

Continued...

10. A computer software developer would like to use the number of downloads (in thousands) for the trial version of his new software to predict the amount of revenue (in thousands of dollars) on selling the full version of his new software. A table is provided as below to display an output from a simple linear regression analysis:

Multiple R	0.8691
R Square	0.7554
Adjusted R Square	0.7467
Standard Error	44.4765
Observations	30

Which of the following is the correct interpretation for the coefficient of determination?

A. 86.91% of the variation in revenue can be explained by the variation in the number of downloads.
B. 75.54% of the variation in revenue can be explained by the variation in the number of downloads.
C. 75.54% of the variation in the number of downloads can be explained by the variation in revenue.
D. 74.67% of the variation in the number of downloads can be explained by the variation in revenue.

Continued...

SECTION B: STRUCTURED QUESTIONS (80 MARKS)

There are THREE questions in this section. Candidates MUST answer ALL THREE questions.

Question 1 (25 Marks)

A bank with a branch located in a commercial district of a city would like to improve the process of serving customers during the noon-to-1p.m. lunch period. Management decides to study the waiting time in the current process. Data are collected from a random sample of 15 customers, and the waiting time (in minutes) are recorded as follows:

4.21	5.55	3.02	5.13	4.77	2.34	3.54	3.20
4.50	6.10	0.38	5.12	6.46	6.19	3.79	

Suppose that another branch located in a residential area, is also concerned to improve the process of serving customers during lunch period. Another random sample of 15 customers and their waiting time (in minutes) are recorded as follows:

9.66	5.90	8.02	5.79	8.73	3.82	8.01	8.35
10.49	6.68	5.64	4.08	6.17	9.91	5.47	

a) What is the average waiting time (in minutes) for both bank branches? Discuss the difference of the average values. [5 marks]

b) Find the median and standard deviation of waiting time (in minutes) for both bank branches. [8 marks]

c) Describe the shape of the distribution of waiting time for both bank branches. [2 marks]

d) At 5 percent significance level, can we conclude that the customers at branch which located in residential area need to wait longer compared to the customers who entered bank branch in the commercial district? [10 marks]

Continued...

Question 2 (25 Marks)

a) Hannah is going to play one badminton match and one tennis match. The badminton match has been scheduled one week earlier than tennis match. Based on the prediction by her coach, the chance that she will win the badminton match is $9/10$. Other than that, the coach believed that if Hannah wins the badminton match, the chance that she will win the tennis match is $3/5$. But if she loses the badminton match, she has $2/3$ chance of losing the tennis match too.

(i) Draw a tree diagram and the probabilities for each events involved for the above scenario. [6 marks]

(ii) Find the probability that Hannah will win for the tennis match. [3 marks]

(iii) The matches have been re-scheduled. Given that Hannah won the tennis match, what is the probability that she will also win the badminton match? [3 marks]

b) According to the 2016 population survey conducted by Statistics Department of Malaysia, 32% of the Malaysia population with age 25 years old or over has completed a bachelor' degree or more. Given a random sample of 10 people with age 25 years old or over,

(i) What is the probability that at most 2 of the selected people has completed a bachelors degree or more? [4 marks]

(ii) What is the average number of people who have completed a bachelors degree or more from the sample?. [2 marks]

c) A test has been devised to measure student's level of motivation during high school. The higher the score the greater the motivation to do well at school. The motivation scores on the test are approximately normally distributed with a mean of 25 and a standard deviation of 6.

(i) What is the probability that a student taking this test will obtain scores between 20 to 28? [4 marks]

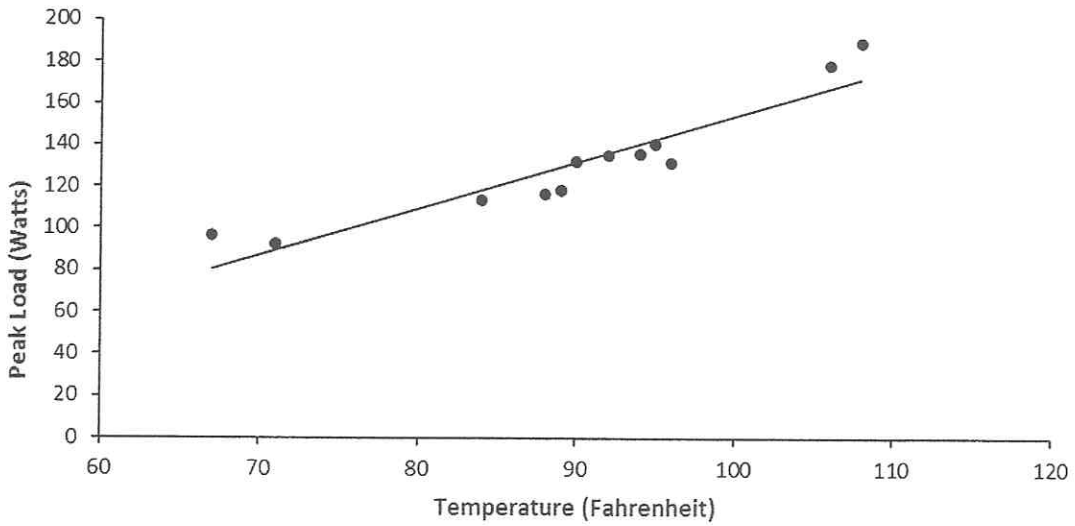
(ii) Eliya has been told that 35% of the students taking the test have higher motivation scores than she does. What is Eliya's score? [3 marks]

Continued...

Question 3 (30 Marks)

a) Peak power load is the maximum amount of power which need to be generate daily to meet customer's demand. A power company wants to use daily high temperature to predict daily peak power load during the summer season when demand is greatest. These data, scatterplot and summary output of the relationship between the two variables are shown below:

Temperature (Fahrenheit)	Peak Load (Watts)
94	136
96	131.7
95	140.7
108	189.3
67	96.5
88	116.4
89	118.5
84	113.4
90	132
106	178.2
71	92.5
92	135.1



Continued...

Summary Output

Regression Statistics	
Multiple R	0.9342
R Square	0.8728
Adjusted R Square	0.8600
Standard Error	10.7661
Observations	12

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	7950.967864	7950.968	68.59715	8.67897E-06
Residual	10	1159.081302	115.9081		
Total	11	9110.049167			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-69.4402	24.4825	-2.83632	0.017659
Temperature	2.2348	0.2698	8.28234	8.68E-06

(i) State the dependent variable and the independent variable for the above regression model. [2 marks]

(ii) Write the least square regression line for the above relationship between the two variables. State the unit of measurement for each variable. [4 marks]

(iii) What do the coefficient of the regression line tell you about the relationship between peak power load and the temperature during summer? [3 marks]

(iv) Determine the coefficient of correlation and discuss the role of this coefficient value for this model. [3 marks]

(v) Predict the peak power load if the temperature is set at 110 Fahrenheit. Is this estimation reliable? Explain. [4 marks]

(iv) State the coefficient of determination and describe what it tells you. [4 marks]

(b) A table below is consisting selected consumer goods for 2003 and 2013:

Product	2003		2013	
	Price (RM)	Quantity	Price (RM)	Quantity
Food	340	115	345	90
Rent	600	16	600	13
Drinks	120	90	150	120
Transportation	250	43	260	56

Compute and interpret the Laspeyres Price Index (LPI) and Paashe Price Index (PPI) for 2013 using 2003 as the base period. [10 marks]

THE END

STATISTICAL FORMULAE

A. DESCRIPTIVE STATISTICS

Mean $(\bar{x}) = \dfrac{\sum_{i=1}^{n} X_i}{n}$

Standard Deviation $(s) = \sqrt{\dfrac{\sum_{i=1}^{n} X_i^{\,2}}{n-1} - \dfrac{(\sum_{i=1}^{n} X_i)^2}{n(n-1)}}$

Coefficient of Variation $(CV) = \dfrac{\sigma}{\bar{X}} \times 100$

Pearson's Coefficient of Skewness $(S_k) = \dfrac{3(\bar{X} - Median)}{s}$

B. PROBABILITY

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A)× P (B) if A and B are independent

P (A | B) = P (A and B) ÷ P (B)

Poisson Probability Distribution

If X follows a Poisson Distribution , $P(\lambda)$ where $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

then the mean = $E(X) = \lambda$ and variance = $VAR(X) = \lambda$

Binomial Probability Distribution

If X follows a Binomial Distribution B(n, p) where $P(X = x) = {}^nC_x p^x q^{n-x}$

then the mean = $E(X) = np$ and variance = $VAR(X) = npq$ where $q = 1 - p$

Normal Distribution

If X follows a Normal distribution, $N(\mu, \sigma)$ where $E(X) = \mu$ and $VAR(X) = \sigma^2$

then $Z = \dfrac{X - \mu}{\sigma}$

C. EXPECTATION AND VARIANCE OPERATORS

$E(X) = \sum[X \bullet P(X)]$

$VAR(X) = E(X^2) - [E(X)]^2$ where $E(X^2) = \sum[X^2 \bullet P(X)]$

If $E(X) = \mu$ then $E(cX) = c\,\mu$, $E(X_1 + X_2) = E(X_1) + E(X_2)$

If $VAR(X) = \sigma^2$ then $VAR(cX) = c^2\,\sigma^2$,

$VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2) + 2\,COV(X_1, X_2)$

where $COV(X_1, X_2) = E(X_1 X_2) - [E(X_1)\,E(X_2)]$

D. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

(100 – α) % Confidence Interval for Population Mean (σ Known) = $\mu = \overline{X} \pm Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Mean (σ Unknown) = $\mu = \overline{X} \pm t_{\alpha/2,n-1}\left(s/\sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Proportion $= \hat{p} \pm Z_{\alpha/2}\sigma_{\hat{p}}$

Where $\sigma_{\hat{p}} = \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Sample Size Determination for Population Mean $= n \geq \left[\dfrac{(Z_{\alpha/2})\sigma}{E}\right]^2$

Sample Size Determination for Population Proportion $= n \geq \dfrac{(Z_{\alpha/2})^2\,\hat{p}(1-\hat{p})}{E^2}$

Where E = Limit of Error in Estimation

E. HYPOTHESIS TESTING

One Sample Mean Test

Standard Deviation (σ) Known	**Standard Deviation (σ) Not Known**
$Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	$t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

One Sample Proportion Test

$z = \dfrac{\hat{p} - p}{\sigma_p}$ where $\sigma_p = \sqrt{\dfrac{p(1-p)}{n}}$

Two Sample Mean Test

Standard Deviation (σ) Known	**Standard Deviation (σ) Not Known**
$z = \dfrac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$	$t = \dfrac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$ where $S_p^2 = \dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1 + n_2 - 2)}$

Two Sample Proportion Test

$Z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $p = \dfrac{X_1 + X_2}{n_1 + n_2}$

where X_1 and X_2 are the number of successes from each population

F. REGRESSION ANALYSIS

Simple Linear Regression

Population Model: $Y = \beta_0 + \beta_1 X_1 + \varepsilon$

Sample Model: $\hat{y} = b_0 + b_1 x_1 + e$

Correlation Coefficient

$$r = \frac{\sum XY - \left[\frac{\sum X \sum Y}{n}\right]}{\sqrt{\left[\sum X^2 - \left(\frac{(\sum X)^2}{n}\right)\right]\left[\sum Y^2 - \left(\frac{(\sum Y)^2}{n}\right)\right]}} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

ANOVA Table for Regression

Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	1	SSR	MSR = SSR/1
Error/Residual	$n-2$	SSE	MSE = SSE/$(n-2)$
Total	$n-1$	SST	

Test Statistic for Significance of the Predictor Variable

$$t_i = \frac{b_i}{S_{b_i}} \quad \text{and the critical value} = \pm t_{\alpha/2,(n-p-1)}$$

Where p= number of predictor

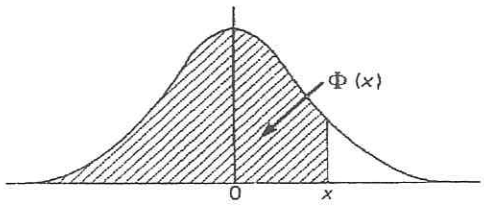
G. INDEX NUMBERS

Simple Price Index $P = \frac{p_t}{p_0} \times 100$	**Laspeyres Quantity Index** $P = \frac{\sum p_0 q_t}{\sum p_0 q_0} \times 100$
Aggregate Price Index $P = \frac{\sum p_t}{\sum p_0}(100)$	**Paasche Quantity Index** $P = \frac{\sum p_t q_t}{\sum p_t q_0} \times 100$
Laspeyres Price Index $P = \frac{\sum p_t q_0}{\sum p_0 q_0} \times 100$	**Fisher's Ideal Price Index** $\sqrt{(Laspeyres\ \ Price\ Index)(Paa\ sche\ Price\ Index)}$
Paasche Price Index $P = \frac{\sum p_t q_t}{\sum p_0 q_t} \times 100$	**Value Index** $V = \frac{\sum p_t q_t}{\sum p_0 q_0} \times 100$

B. STATISTICAL TABLE

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

3·075	0·9990	3·263	0·9994	3·731	0·99990	3·916	0·99995
3·105	0·9991	3·320	0·9995	3·759	0·99991	3·976	0·99996
3·138	0·9992	3·389	0·9996	3·791	0·99992	4·055	0·99997
3·174	0·9993	3·480	0·9997	3·826	0·99993	4·173	0·99998
3·215	0·9994	3·615	0·9998	3·867	0·99994	4·417	0·99999
			0·9999		0·99995		1·00000

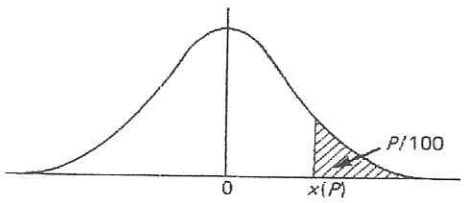
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



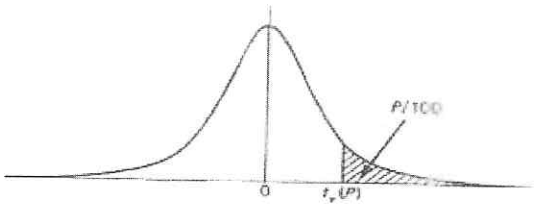
P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

TABLE 10. PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P	40	30	25	20	15	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·3249	0·7265	1·0000	1·3764	1·963	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	0·2887	0·6172	0·8165	1·0607	1·386	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	0·2767	0·5844	0·7649	0·9785	1·250	1·638	2·353	3·182	4·541	5·841	10·21	12·92
4	0·2707	0·5686	0·7407	0·9410	1·190	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	0·2672	0·5594	0·7267	0·9195	1·156	1·476	2·015	2·571	3·365	4·032	5·893	6·869
6	0·2648	0·5534	0·7176	0·9057	1·134	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	0·2632	0·5491	0·7111	0·8960	1·119	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	0·2619	0·5459	0·7064	0·8889	1·108	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	0·2610	0·5435	0·7027	0·8834	1·100	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	0·2602	0·5415	0·6998	0·8791	1·093	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	0·2596	0·5399	0·6974	0·8755	1·088	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	0·2590	0·5386	0·6955	0·8726	1·083	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	0·2586	0·5375	0·6938	0·8702	1·079	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	0·2582	0·5366	0·6924	0·8681	1·076	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	0·2579	0·5357	0·6912	0·8662	1·074	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	0·2576	0·5350	0·6901	0·8647	1·071	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	0·2573	0·5344	0·6892	0·8633	1·069	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	0·2571	0·5338	0·6884	0·8620	1·067	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	0·2569	0·5333	0·6876	0·8610	1·066	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	0·2567	0·5329	0·6870	0·8600	1·064	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	0·2566	0·5325	0·6864	0·8591	1·063	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	0·2564	0·5321	0·6858	0·8583	1·061	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	0·2563	0·5317	0·6853	0·8575	1·060	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	0·2562	0·5314	0·6848	0·8569	1·059	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	0·2561	0·5312	0·6844	0·8562	1·058	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	0·2560	0·5309	0·6840	0·8557	1·058	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	0·2559	0·5306	0·6837	0·8551	1·057	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	0·2558	0·5304	0·6834	0·8546	1·056	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	0·2557	0·5302	0·6830	0·8542	1·055	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	0·2556	0·5300	0·6828	0·8538	1·055	1·310	1·697	2·042	2·457	2·750	3·385	3·646
32	0·2555	0·5297	0·6822	0·8530	1·054	1·309	1·694	2·037	2·449	2·738	3·365	3·622
34	0·2553	0·5294	0·6818	0·8523	1·052	1·307	1·691	2·032	2·441	2·728	3·348	3·601
36	0·2552	0·5291	0·6814	0·8517	1·052	1·306	1·688	2·028	2·434	2·719	3·333	3·582
38	0·2551	0·5288	0·6810	0·8512	1·051	1·304	1·686	2·024	2·429	2·712	3·319	3·566
40	0·2550	0·5286	0·6807	0·8507	1·050	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	0·2547	0·5278	0·6794	0·8489	1·047	1·299	1·676	2·009	2·403	2·678	3·261	3·496
60	0·2545	0·5272	0·6786	0·8477	1·045	1·296	1·671	2·000	2·390	2·660	3·232	3·460
120	0·2539	0·5258	0·6765	0·8446	1·041	1·289	1·658	1·980	2·358	2·617	3·160	3·373
∞	0·2533	0·5244	0·6745	0·8416	1·036	1·282	1·645	1·960	2·326	2·576	3·090	3·291